profile for tests conducted by Bendix Corporation[2] are shown in Fig 2 As can be seen from the figures, the analytical results agree favorably with experimental evidence It should be pointed out that in *no* instance were the analytical results *optimistic* as claimed by Lavender

The question of the discontinuity in the stability profile presented by the Marshall Space Flight Center computer program is of concern to the writer No such jump in the stability profile was obtained in the outlined analytical and experimental programs A continuous plot of the vehicle motion and impact force histories was obtained when the problem was formulated on the analog computer The results of this study are reported in Ref 3 The free-flight condition described by Lavender was obtained in both the digital and analog programs, but no discontinuity in the stability profile was observed It is unlikely that this condition explains the discontinuity in the Marshall Space Flight Center results Since the analytical procedure or mathematical model used are not described by Lavender, it is difficult to speculate as to the cause of the discontinuity

It should also be mentioned that the basic analysis was proposed for the consideration of the three-dimensional alightment problem

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[3] Cappelli A P Parametric studies of the landing stability of lunar alightment vehicles," STR 79, Space & Information Div , North American Aviation, Inc (1961)

Some Effects of Planform Modification on the Skin Friction Drag

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For the rapid estimation of the skin friction drag of airplane wing and control surfaces, it is sometimes the practice to use the Reynolds number based on the average chord of the surface in the calculations This note will show that, if a simple average geometric chord is used, such an approximate solution can differ significantly from the more exact solution involving a spanwise integration of the skin friction

The general planform of a half-wing to be analyzed is shown in Fig 1 with some of the symbols that will be used For a two-dimensional wing the aveıage skin-friction coefficient can be represented by

$$C_F = K(R/l)^n c^n \tag{1}$$

where

n = exponent of skin-friction equation
K = constant of skin-friction equation (considered herein to include compressibility and heat-trans fer effects)
R/l = unit Reynolds number

When the average skin friction obtained by Eq (1) at each spanwise station is integrated over the half-wing span s and the result is divided by the dynamic pressure and by the

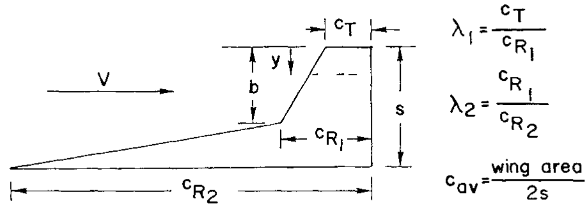

Fig 1 Geometry and symbols

plan area of the half-wing, we obtain a general expression for the skin-friction coefficient:

$$C_F = \frac{K(R/l)\, 2^{n+1}(c_{av})^n}{n+2} \times \left\{\frac{\lambda_2}{(1+\lambda_2)[1-(b/s)] + (1+\lambda_1)(\lambda_2)(b/s)}\right\}^{n+1} \times \left\{\left[\frac{1-(\lambda_1)^{n+2}}{1-\lambda_1}\right]\frac{b}{s} + \left(1-\frac{b}{s}\right)\left[\frac{1-(\lambda_2)^{n+2}}{(1-\lambda_2)(\lambda_2)^{n+1}}\right]\right\} \tag{2}$$

for which the following limits apply:

$$0 \le \lambda_1 \le 1 \qquad 0 < \lambda_2 \le 1 \qquad 0 \le b/s < 1$$

where, by L'Hospital's rule,

$$\lim_{\lambda_1 \to 1}\left[\frac{1-(\lambda_1)^{n+2}}{1-\lambda_1}\right] = \lim_{\lambda_2 \to 1}\left[\frac{1-(\lambda_2)^{n+2}}{(1-\lambda_2)(\lambda_2)^{n+1}}\right] = n+2$$

When there are no restraints on the factors contained in Eq (2), the skin friction approaches 0 as $\lambda_2 \to 0$, obviously an impractical solution A family of planforms is analyzed, therefore, which contains wings having the same ratio of average chord to root chord To facilitate such an analysis, the notch ratio b/s in Eq (2) will be written as

$$\frac{b}{s} = \frac{1+\lambda_2-(2c_{av}/c_{R_2})}{1-\lambda_1\lambda_2} \tag{3}$$

All wings will be assumed to be flying at the same unit Reynolds number and Mach number and to have the same average chord, span, and area If we consider only wings with $\lambda_1 = 0$, then from Eqs (2) and (3) we can write the equation of the skin-friction coefficient of the cranked wing to that for the untapered wing as

$$\frac{(C_F)_{\lambda_2, \lambda_1=0}}{(C_F)_{\lambda_1=\lambda_2=b/s=1}} = \frac{2^{n+1}}{n+2}\left(\frac{\lambda_2}{2c_{\ v}/c_{R_2}}\right)^{n+1}\left\{1+\lambda_2-\frac{2c_{av}}{c_{R_2}} + \left(\frac{2c_{av}}{c_{R_2}}-\lambda_2\right)\left[\frac{1-(\lambda_2)^{n+2}}{(1-\lambda_2)(\lambda_2)^{n+2}}\right]\right\} \tag{4}$$

The results of varying the inboard taper ratio λ_2 in Eq (4) on the skin friction of cranked wings to that for a rectangular wing are presented in Fig 2 for an assumed $2c_{\ v}/c_{R_2} = \frac{1}{2}$ for both the turbulent ($n = -\frac{1}{7}$ for large Reynolds numbers, see Ref 1) and the laminar ($n = -\frac{1}{2}$) boundary-layer cases Wing planforms are also shown in Fig 2 above each corresponding taper ratio For the conditions specified, it is apparent that using the average chord in calculating the skin friction of such planforms can lead to sizable errors

Wings having straight leading edges (no crank) will be con sidered next This case can easily be derived from Eq (2) by letting $b/s = \lambda_2 = 1\ 0$ Provided the same conditions are imposed on the tapered and the untapered wings, namely, that each wing has the same average chord, span, area, unit Reynolds numbers, and Mach number, then the skin-friction ratio for the tapered wing to that for the rectangular wing be comes

$$\frac{(C_F)_{\lambda_1}}{(C_F)_{\lambda_1=1}} = \frac{2^{n+1}}{n+2}\left(\frac{1}{1+\lambda_1}\right)^{n+1}\left[\frac{1-(\lambda_1)^{n+2}}{1-\lambda_1}\right] \tag{5}$$

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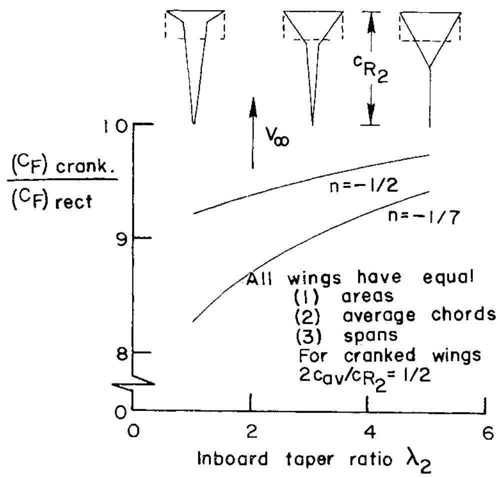

Fig 2 Ratios of the average skin-friction coefficient for cranked wings to rectangular wings

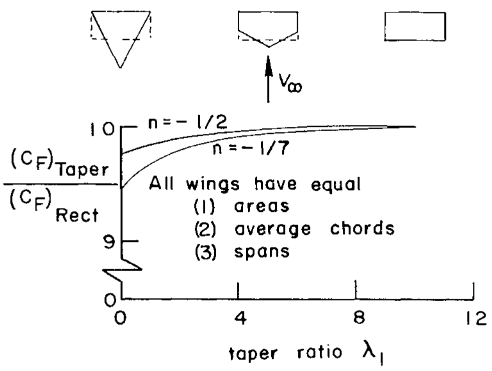

Fig 3 Ratios of average skin-friction coefficient for tapered wings to rectangular wings

for the limit $0 \leq \lambda_1 \leq 1$ where again

$$\lim_{\lambda_1 \to 1}\left[\frac{1-(\lambda_1)^{n+2}}{1-\lambda_1}\right] = n + 2$$

Effects of taper ratio variations on the foregoing skin-friction ratio are shown in Fig 3 The results indicate that for wings having straight leading edges and small taper ratios the skin friction cannot be obtained accurately from a Reynolds number based on its average chord

Reference

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Comment on "Velocity Defect Law for a Transpired Turbulent Boundary Layer"

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IN their recent contribution to this Journal, Mickley and Smith[1] explain the behavior of transpired turbulent boundary layers in a novel way Former work of Mickley[2] was based on Rubesin's[3] application of the mixture-length theory to turbulent boundary layers with suction or injection

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This theory predicts a bilogarithmic mean velocity profile In contrast to this, a semilogarithmic mean velocity profile is characteristic of Clauser's[4] phenomelogical description of turbulent boundary layers It appears that Mickley and Smith abandoned the former theory in favor of the latter Their changed approach agrees with the development at our laboratory The results of a rather large number of experiments forced us to accept that turbulent boundary layers with distributed suction have semilogarithmic mean velocity profiles, without any positive tendency towards conformity with the bilogarithmic law The slope of the logarithmic part of the mean velocity profile appears to depend on the ratio v_0/u_τ only, as Fig 1 shows

The data, which exhibit rather much scatter because of the inaccuracy of determining the slope from experimental curves, suggest the following relation:

$$w^* = x_2(\partial U_1/\partial x_2) = 2\ 3\ u_\tau(1 + 9v_0/u_\tau) \qquad (1)$$

It is proposed to call w^* the "logarithmic velocity scale" The relation suggested by Mickley and Smith, i e , that the logarithmic velocity scale (equivalent to $U_T{}^*$ in their notation) is proportional to the square root of the maximum Reynolds stress within the boundary layer, cannot be applied to aspired turbulent boundary layers For these layers the shear stress attains its maximum in a sharp peak at the wall, which is not representative for the level of Reynolds stress at the outer edge of the inner layer, as experiments performed by Favre et al [5] have shown

The empirical relation (1), however, is not very well suited to describe the behavior of turbulent boundary layers with moderate suction ($0\ 04 < -v_0/u_\tau < 0\ 10$) These layers have a relatively thick viscous sublayer which is described by

$$\frac{v_0 U_1}{u_\tau^2} = \exp\frac{v_0 x_2}{\nu} - 1 \qquad (2)$$

This equation, which gives the mean velocity in the lowermost part of the "inner layer," suggests the following general relation for the mean velocity in the inner layer:

$$\frac{v_0 U_1}{u_\tau^2} = f_1\left(\frac{v_0 x_2}{\nu}\right) \qquad (3)$$

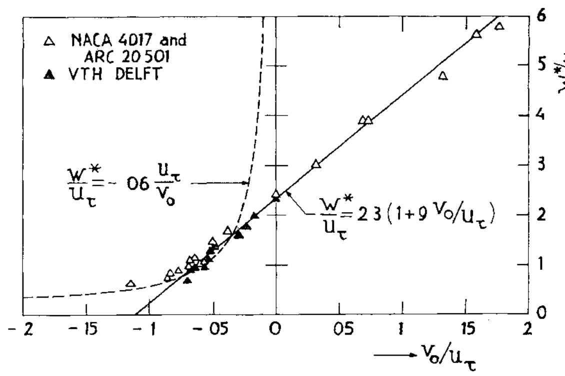

Fig 1 The logarithmic velocity scale

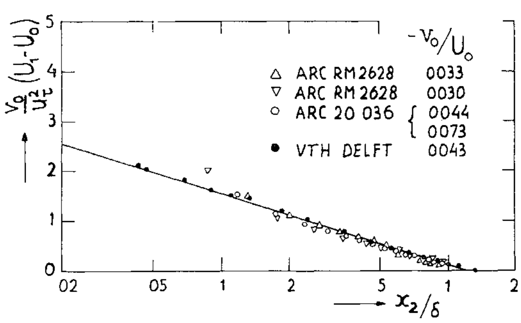

Fig 2 Mean velocity profiles of asymptotic layers